

Inventory Turnover Ratio Model For Some Important Mathematical Cases using Stock dependent consumption rate when setup cost is fixed.

Principal

Dr. Sudhir G. Joshi

Shri H. S. Shah College of Commerce,
Modasa – 383 315

Email: joshisudhirg@yahoo.co.in

Abstract:

Ratio Analysis is one of the most important tool for financial analysis. There for it is important to obtain some meaningful information such as liquidity, profitability efficiency and financial position of a company by using ratio as a tool. Financial ratios can be classified under the following five categories (i) Structural (ii) Liquidity (iii) Profitability (iv) Turnover (v) Miscellaneous group. Inventory turnover ratio is a broad measure. It does not bring into light several important facts. For example, high inventory turnover ratio might lead the people to believe that the management is handling greater sales with less investment in inventory. However, it fails to reveal the amount of sales lost by the company owing to maintenance of low inventories or even periodic stock outs, lost incurred as a result of frequent reordering or production runs, cost of storing large accumulations of inventory out of large production run. It would be therefore be genuine to evolve such techniques as may evaluate the management of inventory in the light of objectives of inventory management. The most important objective of inventory management is to minimize risk of being out of stock and to reduce cost of obsolescence and spoilage and other carrying costs. The management must, therefore, establish some mechanisms and systems to check the firm's performance in above respects. In this paper, we have try to fit Inventory Turnvover Ratio (ITOR) aspect of Financial Accountiung Management under Economic Order Quantity approach.

Key Words : Ratio Analysis, ITOR, EOQ, Holding Cost, Setup Cost, Demand, Lot Size

Introduction:

The inventory turn over ratio discussed below under a stock dependent consumption rate.

The inventory turnover ratio model was proposed by S.B.Srivastav(1978) and supported by Kanti Swaroop-P.K.Gupta & Man Mohan(1994)

Notations:

C_1 = Inventory Carrying / Holding Cost per Rs. Per Unit per Year

C_3 = Ordering Cost / Setup Cost per Rs. per Order / Production

D = Demand per Unit per Year

Q = Lot Size / Contract quantity

P = Purchase/Selling Price per Unit

Assumptions:

- (i) Demand is stock dependent.
- (ii) Shortages are not allowed.
- (iii) Lead-time is zero.
- (iv) Replenishment Rate is infinite.

Problem Formulation:

The Inventory Turnover Ratio is defined as

$$I(Q) = \frac{DP}{C(Q)} \quad \dots\dots 1$$

$$\left[\frac{QC_1}{2} + \frac{DC_3}{Q} \right]$$

In order to find the optimum values for Q so as to maximize Inventory Turnover Ratio, we have,

$$\begin{aligned} \frac{\partial I(Q)}{\partial Q} &= \frac{\partial}{\partial Q} \left[DP \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^{-1} \right] \\ &= -DP \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^{-2} \left(\frac{C_1}{2} - \frac{DC_3}{Q^2} \right) \\ \text{but } \frac{\partial I(Q)}{\partial Q} &= 0 \\ \therefore -DP \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^{-2} \left(\frac{C_1}{2} - \frac{DC_3}{Q^2} \right) &= 0 \end{aligned}$$

$$\therefore \frac{C_1}{2} - \frac{DC_3}{Q^2} = 0$$

$$\therefore Q = \sqrt{\frac{2DC_3}{C_1}}$$

The Inventory Turnover Ratio is maximum only if $\frac{\partial^2 I(Q)}{\partial Q^2} < 0$. Therefore,

$$\begin{aligned} \frac{\partial^2 I(Q)}{\partial Q^2} &= \frac{\partial}{\partial Q} \left(\frac{\partial I(Q)}{\partial Q} \right) \\ &= \frac{\partial}{\partial Q} \left\{ -DP \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^{-2} \left(\frac{C_1}{2} - \frac{DC_3}{Q^2} \right) \right\} \\ &= - \left\{ DP \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^{-2} \left(\frac{2DC_3}{Q^3} \right) - 2DP \left(\frac{C_1}{2} - \frac{DC_3}{Q^2} \right) \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^{-3} \left(\frac{C_1}{2} - \frac{DC_3}{Q^2} \right) \right\} \\ &= - \left\{ \frac{2D^2PC_3}{Q^3 \left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^2} - \frac{2DP \left(\frac{C_1}{2} - \frac{DC_3}{Q^2} \right)^2}{\left(\frac{QC_1}{2} + \frac{DC_3}{Q} \right)^3} \right\} \\ &< 0 \end{aligned}$$

The optimum value of Q has thus been obtained and is given by,

$$Q_{opt} = \sqrt{\frac{2DC_3}{C_1}} \quad \dots\dots 2$$

and optimum Inventory Turnover Ratio is given by

$$\begin{aligned} I(Q_{opt}) &= \frac{DP}{\left[\frac{Q_{opt}C_1}{2} + \frac{DC_3}{Q_{opt}} \right]} \\ &= \frac{DP}{\left[\frac{C_1}{2} \sqrt{\frac{2DC_3}{C_1}} + DC_3 \sqrt{\frac{C_1}{2DC_3}} \right]} \\ &= \frac{DP}{\left[\sqrt{\frac{C_1C_3D}{2}} + \sqrt{\frac{C_1C_3D}{2}} \right]} \end{aligned}$$

$$= \frac{DP}{2\sqrt{\frac{C_1 C_3 D}{2}}}$$

$$= DP / \sqrt{2C_1 C_3 D} \quad \text{or} \quad \frac{P\sqrt{D}}{\sqrt{2C_1 C_3}} \quad \dots\dots 3$$

Hypothetical Problem:-

Item	Demand (Units) D	Holding Cost C ₁	Ordering Cost C ₃	Purchase Cost P	Optimal Turnover (Units) Q _{opt}	Total Inventory Cost C(Q _{opt})	Optimal Turnover Ratio I(Q _{opt})
1	8,000	Rs. 25	Rs. 70	Rs. 5	212	Rs.5292	7.56
2	9,500	Rs. 25	Rs. 60	Rs. 6	214	Rs.5339	10.68
3	12,000	Rs. 25	Rs. 50	Rs. 7	219	Rs.5477	15.34
4	13,500	Rs. 25	Rs. 45	Rs. 7.5	220	Rs.5511	18.37
5	15,000	Rs. 25	Rs. 40	Rs. 8	219	Rs.5477	21.91
6	18,500	Rs. 25	Rs. 30	Rs. 9	211	Rs.5268	31.61

Note: Above hypothetical problem is solved by C++ Program mentioned in appendix 1. Necessary graphical representation is also mentioned in appendix 2.

Remark:-

It is clear from above example that optimal turnover is depend on consuption rate. The size of turnover increases upto particular level to maintaining optimal ordering cost. For a given demand, optimal turnover increases the turnover ratio increases, it means there are no shortages of inventories and management of handling the inventory is efficient in the firm. So, it is beneficial for the firm.

Case-I: $D=\alpha+\beta Q$

Demand D is a linear function of lot size Q. Let us take $D=\alpha+\beta Q$, Where α and β are constants and the Inventory Turnover Ratio is given by,

$$I(Q) = DP / C(Q)$$

$$= \frac{DP}{\left[\frac{QC_1}{2} + \frac{DC_3}{Q} \right]}$$

$$\begin{aligned}
 &= P(\alpha + \beta Q) \left[\frac{QC_1}{2} + \frac{(\alpha + \beta Q)C_3}{Q} \right]^{-1} \\
 &= (\alpha P + \beta PQ) \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-1} \quad \dots\dots 4
 \end{aligned}$$

To obtain the optimum value of Q, nessosary and sufficient conditions are

$$\frac{\partial I(Q)}{\partial Q} = 0 \text{ and } \frac{\partial^2 I(Q)}{\partial Q^2}, \text{ thus,}$$

$$\begin{aligned}
 \frac{\partial I(Q)}{\partial Q} &= (O + \beta P) \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-1} + (-1)(\alpha P + \beta PQ) \\
 &\quad \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-2} \left[\frac{C_1}{2} - \frac{\alpha C_3}{Q^2} + 0 \right]
 \end{aligned}$$

$$\text{but } \frac{\partial I(Q)}{\partial Q} = 0$$

$$\begin{aligned}
 \therefore \beta P - (\alpha P + \beta PQ) \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-1} \left[\frac{C_1}{2} - \frac{\alpha C_3}{Q^2} \right] &= 0 \\
 \therefore \beta P \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right] - (\alpha P + \beta PQ) \left[\frac{C_1}{2} - \frac{\alpha C_3}{Q^2} \right] &= 0 \\
 \therefore \beta P \left[\frac{Q^2 C_1 + 2\alpha C_3 + 2\beta C_3 Q}{2Q} \right] - (\alpha P + \beta PQ) \left[\frac{C_1 Q^2 - 2\alpha C_3}{2Q^2} \right] &= 0 \\
 \therefore \beta PQ [Q^2 C_1 + 2\alpha C_3 + 2\beta C_3 Q] - (\alpha P + \beta PQ) [C_1 Q^2 - 2\alpha C_3] &= 0 \\
 \therefore \beta PC_1 Q^3 + 2\alpha \beta C_3 PQ + 2\beta^2 PC_3 Q^2 - \alpha PC_1 Q^2 - \beta PC_1 Q^3 + 2\alpha^2 PC_3 + 2\alpha \beta PC_3 Q &= 0 \\
 \therefore (2\beta^2 PC_3 - \alpha PC_1) Q^2 + 4\alpha \beta PC_3 Q + 2\alpha^2 PC_3 &= 0 \quad \dots\dots 5
 \end{aligned}$$

$$\text{Now, } Q = \frac{-b \pm \sqrt{\Delta}}{2a} \quad \text{where } \Delta = b^2 - 4ac$$

$$\begin{aligned}
 \therefore \Delta &= 16\beta^2 C_3^2 P^2 \alpha^2 - 4(2\beta^2 PC_3 - \alpha PC_1) 2\alpha^2 PC_3 \\
 &= 16\alpha^2 \beta^2 P^2 C_3^2 - 16\alpha^2 \beta^2 P^2 C_3^2 + 8\alpha^3 P^2 C_1 C_3 \\
 &= 8\alpha^3 P^2 C_1 C_3
 \end{aligned}$$

$$\begin{aligned}
 \therefore Q &= \frac{-b \pm \sqrt{\Delta}}{2a} \\
 &= \frac{-4\alpha \beta PC_3 \pm \sqrt{8\alpha^3 P^2 C_1 C_3}}{2(2\beta^2 PC_3 - \alpha PC_1)}
 \end{aligned}$$

$$\text{and } \frac{\partial^2 I(Q)}{\partial Q^2} = 0 - \beta P \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-1} \left[\frac{C_1}{2} - \frac{\alpha C_3}{Q^2} \right] + (-1)(\alpha P + \beta P Q) \left(\frac{C_1}{2} - \frac{\alpha C_3}{Q^2} \right) \\ \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-2} \left[\frac{C_1}{2} - \frac{\alpha C_3}{Q^2} \right] + \beta P \left[\frac{QC_1}{2} + \frac{\alpha C_3}{Q} + \beta C_3 \right]^{-1} \left(\frac{2\alpha C_3}{Q^3} \right) \\ < 0 \\ \therefore Q_{opt} = \frac{-4\alpha\beta PC_3 \pm \sqrt{8\alpha^3 P^2 C_1 C_3}}{4\beta^2 PC_3 - 2\alpha PC_1} \quad \dots\dots 6$$

Which maximize the inventory turnover ratio $I(Q)$.

Therefore,

$$I\left(\frac{Q}{opt}\right) = \frac{DP}{\left[\frac{Q_{opt} C_1}{2} + \frac{DC_3}{Q_{opt}} \right]} \quad \dots\dots 7$$

Hypothetical Problem:

Item	Demand (Units) $D=\alpha+\beta Q$	Holding Cost (C_1)	Ordering Cost (C_3)	Purchase Cost (p)	α	β	Optimal Turnover (Units) Q_{opt}	Total Inventory Cost $C(Q_{opt})$	ITOR $I(Q_{opt})$
1	1493	Rs. 20	Rs. 60	Rs. 5	200	5	259	Rs.2932	2.55
2	2544	Rs. 20	Rs. 51	Rs. 6	220	6	387	Rs.4209	3.63
3	3018	Rs. 20	Rs. 41.5	Rs. 7	240	7	397	Rs.4285	4.93
4	3621	Rs. 20	Rs. 35	Rs. 8	260	8	420	Rs.4503	6.43
5	4093	Rs. 20	Rs. 30	Rs. 9	280	9	424	Rs.4526	8.14

Note: Above hypothetical problem is solved by C++ Program mentioned in appendix 1. Necessary graphical representation is also mentioned in appendix 2.

Remark:-

It is clear from above example that optimal turnover is depend on α & β . The size of turnover increases maintaining the lower ordering cost. For a linear demand, optimal turnover increases the turnover ratio increases. So, it is beneficial for the company.

Case-II: $D=\beta Q^x$

Let us take demand D as given below:-

$$D = \beta Q^x$$

Where x is a variable, $0 < x < 1$;

β is a Constant

Q is a Contract Quantity

The inventory turnover ratio is given by

$$\begin{aligned}
 I(Q) &= DP \left/ \left[\frac{QC_1}{2} + \frac{DC_3}{Q} \right] \right. \\
 &= P\beta Q^x \left/ \left[\frac{QC_1}{2} + \frac{C_3}{Q} \beta Q^x \right] \right. \\
 &= p\beta Q^x \left[\frac{QC_1}{2} + C_3 \beta Q^{x-1} \right]^{-1} \quad \dots\dots 8
 \end{aligned}$$

The turnover ratio is maximum if (i) $\frac{\partial I(Q)}{\partial Q} = 0$ and $\frac{\partial I(Q)}{\partial Q} < 0$. Thus,

$$\frac{\partial I(Q)}{\partial Q} = x\beta P Q^{x-1} \left[\frac{QC_1}{2} + C_3 \beta Q^{x-1} \right]^{-1} - \beta P Q^x \left[\frac{QC_1}{2} + C_3 \beta Q^{x-1} \right]^{-2} \left[\frac{C_1}{2} + (x-1)C_3 \beta Q^{x-2} \right] \quad \dots\dots 9$$

$$\text{but } \frac{\partial I(Q)}{\partial Q} = 0$$

$$\therefore x\beta P Q^{x-1} \left[\frac{QC_1}{2} + C_3 \beta Q^{x-1} \right]^{-1} - \beta P Q^x \left[\frac{QC_1}{2} + C_3 \beta Q^{x-1} \right]^{-2} \left[\frac{C_1}{2} + (x-1)C_3 \beta Q^{x-2} \right] = 0$$

$$\therefore xQ^{-1} \left[\frac{QC_1}{2} + C_3 \beta Q^{x-1} \right] - \left[\frac{C_1}{2} + (x-1)C_3 \beta Q^{x-2} \right] = 0$$

$$\therefore \frac{x C_1}{2} + x C_3 \beta Q^{x-2} - \frac{C_1}{2} - x C_3 \beta Q^{x-2} + C_3 \beta Q^{x-2} = 0$$

$$\therefore -\frac{C_1}{2}(1-x) + C_3 \beta Q^{x-2} = 0$$

$$\therefore C_3 \beta Q^{x-2} = \frac{C_1}{2}(1-x)$$

$$\therefore Q^{x-2} = \frac{C_1(1-x)}{2C_3\beta}$$

$$\therefore Q^x = \frac{(1-x)C_1 Q^2}{2C_3\beta}$$

$$\therefore Q = \left[\frac{(1-x)C_1 Q^2}{2C_3\beta} \right]^{\frac{1}{x}}$$

or

$$Q = \sqrt[x]{\frac{(1-x)C_1Q^2}{2C_3\beta}} \quad \dots\dots 10$$

and

$$\begin{aligned} \frac{\partial^2 I(Q)}{\partial Q^2} &= \left[\frac{x^2 \beta P C_1 Q^{x-1}}{2} - \frac{x \beta P C_1 Q^{x-1}}{2} + (2x-2) \beta^2 P C_3 Q^{2x-3} \right] \left[\frac{Q C_1}{2} + \beta C_3 Q^{x-1} \right] \\ &\quad - 2 \left[\left(\frac{C_1}{2} + (x-1) \beta C_3 Q^{x-2} \right) \left(\frac{x \beta C_1 Q^x}{2} - \frac{\beta P C_1 Q^3}{2} + \beta^2 P C_3 Q^{2x-3} \right) \right] \\ &\quad \left[\frac{Q C_1}{2} + \beta C_3 Q^{x-1} \right]^{-3} < 0 \end{aligned}$$

Thus 10 becomes,

$$\therefore Q_{opt} = \sqrt[x]{\frac{(1-x)C_1Q^2}{2C_3\beta}} \quad \dots\dots 11$$

Which maximize the inventory turnover ratio $I(Q)$.

Therefore,

$$I\left(\frac{Q}{opt}\right) = \frac{DP}{\left[\frac{Q C_1}{2} + \frac{D C_3}{Q} \right]}$$

Hypothetical Problem:

To understand this ITOR Model, we use some hypothetical value of C_1 , C_3 , x and β and substituting in equation 11, we have,

Item	Demand (Units) $D=\beta Q^x$	Holding Cost (C_1)	Ordering Cost (C_3)	Purchase Cost (P)	X	β	Optimal Turnover (Units)	Total Inventory Cost $C(Q)$	$I(Q_{opt})$
1	3719	Rs.20	Rs.60	Rs.5	0.9	15	485	Rs.5334.18	3.67
2	2476	Rs.20	Rs.50	Rs.6	0.8	30	249	Rs.2985.83	4.98
3	1658	Rs.20	Rs.40	Rs.7	0.7	50	149	Rs.1932.77	6.00
4	985	Rs.20	Rs.35	Rs.8	0.6	65	93	Rs.1299.96	6.06
5	626	Rs.20	Rs.30	Rs.9	0.5	61	61	Rs.919.57	6.13

Note: Above hypothetical problem is solved by C++ Program mentioned in appendix 1. Necessary graphical representation is also mentioned in appendix 2.

Remark:

As β increases and x ($0 < x < 1$) decreases, there is decreases in optimal turnover and ITOR increses and hence the handling of inventory in the firm is at satisfactory level.

Case III: $D=\beta_1Q-\beta_2Q^2$

The Inventory turnover ratio is given by,

$$I(Q) = DP/C(Q)$$

$$\frac{DP}{\left[\frac{QC_1}{2} + \frac{DC_3}{Q}\right]}$$

Let us take demand as $D=\beta_1Q-\beta_2Q^2$

Where β_1 and β_2 are constants and Q is a contract quantity.

$$I(Q) = P(\beta_1Q - \beta_2Q^2) \left/ \left[\frac{QC_1}{2} + \frac{C_3}{Q}(\beta_1Q - \beta_2Q^2) \right] \right.$$

$$= (P\beta_1Q - P\beta_2Q^2) \left[\frac{QC_1}{2} + C_3\beta_1 - C_3\beta_2Q \right]^{-1} \quad \dots\dots 12$$

The turnover ratio is maximum iif (i) $\frac{\partial I(Q)}{\partial Q} = 0$ and (ii) $\frac{\partial^2 I(Q)}{\partial Q^2} < 0$. Thus,

$$\frac{\partial I(Q)}{\partial Q} = (P\beta_1 - 2P\beta_2Q) \left[\frac{QC_1}{2} + C_3\beta_1 - C_3\beta_2Q \right]^{-1} -$$

$$(P\beta_1Q - P\beta_2Q^2) \left(\frac{QC_1}{2} + C_3\beta_1 - C_3\beta_2Q \right)^{-2} \left(\frac{C_1}{2} - C_3\beta_2 \right) \quad \dots\dots 13$$

$$\text{but } \frac{\partial I(Q)}{\partial Q} = 0$$

$$\therefore (P\beta_1 - 2P\beta_2Q) - (P\beta_1Q - P\beta_2Q^2) \left(\frac{C_1}{2} - C_3\beta_2 \right) \left(\frac{QC_1}{2} + C_3\beta_1 - C_3\beta_2Q \right)^{-1} = 0$$

$$\therefore 2(P\beta_1 - 2P\beta_2Q) \left(\frac{QC_1}{2} + C_3\beta_1 - C_3\beta_2Q \right) -$$

$$\begin{aligned}
 & (P\beta_1 C_1 Q - P\beta_2 C_1 Q^2 - 2P\beta_1 \beta_2 C_3 Q + 2P\beta_2^2 C_3 Q^2) = 0 \\
 \therefore & \frac{2P\beta_1 C_1 Q}{2} + 2P\beta_1^2 C_3 - 2P\beta_1 \beta_2 C_3 Q - \frac{4P\beta_2 C_1 Q^2}{2} - 4P\beta_1 \beta_2 C_3 Q + 4P\beta_2^2 C_3 Q^2 \\
 & + P\beta_2 C_1 Q^2 - P\beta_1 C_1 Q + 2P\beta_1 \beta_2 C_3 Q - 2P\beta_2^2 C_3 Q^2 = 0 \\
 \therefore & -P\beta_2 C_1 Q^2 + 2P\beta_2^2 C_3 Q^2 - 4P\beta_1 \beta_2 C_3 Q + 2P\beta_1^2 C_3 = 0 \\
 \therefore & (2P\beta_2^2 C_3 - P\beta_2 C_1) Q^2 - 4P\beta_1 \beta_2 C_3 Q + 2P\beta_1^2 C_3 = 0
 \end{aligned}$$

now $\Delta = b^2 - 4ac$

$$\begin{aligned}
 & = 16P^2 \beta_1^2 \beta_2^2 C_3^2 - 4(2P\beta_2^2 C_3 - P\beta_2 C_1) 2P\beta_1^2 C_3 \\
 & = 16P^2 \beta_1^2 \beta_2^2 C_3^2 - 16P^2 \beta_1^2 \beta_2^2 C_3^2 + 8P^2 \beta_1^2 \beta_2 C_1 C_3 \\
 & = 8P^2 \beta_1^2 \beta_2 C_1 C_3
 \end{aligned}$$

$$\therefore Q = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{4P\beta_1 \beta_2 C_3 \pm \sqrt{8P^2 \beta_1^2 \beta_2 C_1 C_3}}{2(2P\beta_2^2 C_3 - P\beta_2 C_1)}$$

$$= \frac{4P\beta_1 \beta_2 C_3 \pm \sqrt{8P^2 \beta_1^2 \beta_2 C_1 C_3}}{4P\beta_2^2 C_3 - 2P\beta_2 C_1}$$

..... 14

Under certain conditions, it is observed that the turnover ratio is maximum

only if $\frac{\partial^2 I(Q)}{\partial Q^2} < 0$

$$\begin{aligned}
 \therefore \frac{\partial^2 I(Q)}{\partial Q^2} & = -2 \left[\frac{C_1}{2} - \beta_2 C_3 \right] \left[P\beta_1^2 C_3 + \frac{P\beta_2 C_1 Q^2}{2} - 2P\beta_1 \beta_2 C_3 Q \right] \left[\frac{QC_1}{2} + \beta_1 C_3 - \beta_2 C_3 Q \right]^{-3} \\
 & + \left[2P\beta_2^2 C_3 Q - P\beta_2 C_1 Q - 2P\beta_1 \beta_2 C_3 \right] \left[\frac{QC_1}{2} + \beta_1 C_3 - \beta_2 C_3 Q \right]^{-2} < 0
 \end{aligned}$$

$$Q_{opt} = \frac{4P\beta_1 \beta_2 C_3 \pm \sqrt{8P^2 \beta_1^2 \beta_2 C_1 C_3}}{4P\beta_2^2 C_3 - 2P\beta_2 C_1}$$

..... 15

Which maximize the inventory turnover ratio $I(Q)$.

Therefore,

$$I\left(\frac{Q}{opt}\right) = \frac{DP}{\left[\frac{QC_1}{2} + \frac{DC_3}{Q_{opt}} \right]}$$

Hypothetical Problem:

Using hypothetical values, we solve this type of ITOR Model as below:-

Item	Demand	Holding Cost (C_1)	Ordering Cost (C_3)	Purchase Cost (P)	β_1	β_2	Optimal Turnover (units) Q_{opt}	Total Inventory Cost $C(Q_{opt})$	Optimal Turnover Ratio $I(Q_{opt})$
1	2274	Rs. 20	Rs. 60	Rs. 5	70	0.5	89	Rs.2425	4.69
2	3882	Rs. 20	Rs. 50	Rs. 6	80	0.4	117	Rs.2828	8.24
3	8316	Rs. 20	Rs. 40	Rs. 7	100	0.3	174	Rs.3651	15.94
4	17710	Rs. 20	Rs. 30	Rs. 8	120	0.2	262	Rs.4648	30.48
5	41851	Rs. 20	Rs. 20	Rs. 9	140	0.1	433	Rs.6261	60.16

Note: Above hypothetical problem is solved by C++ Program mentioned in appendix 1. Necessary graphical representation is also mentioned in appendix 2.

Remark:

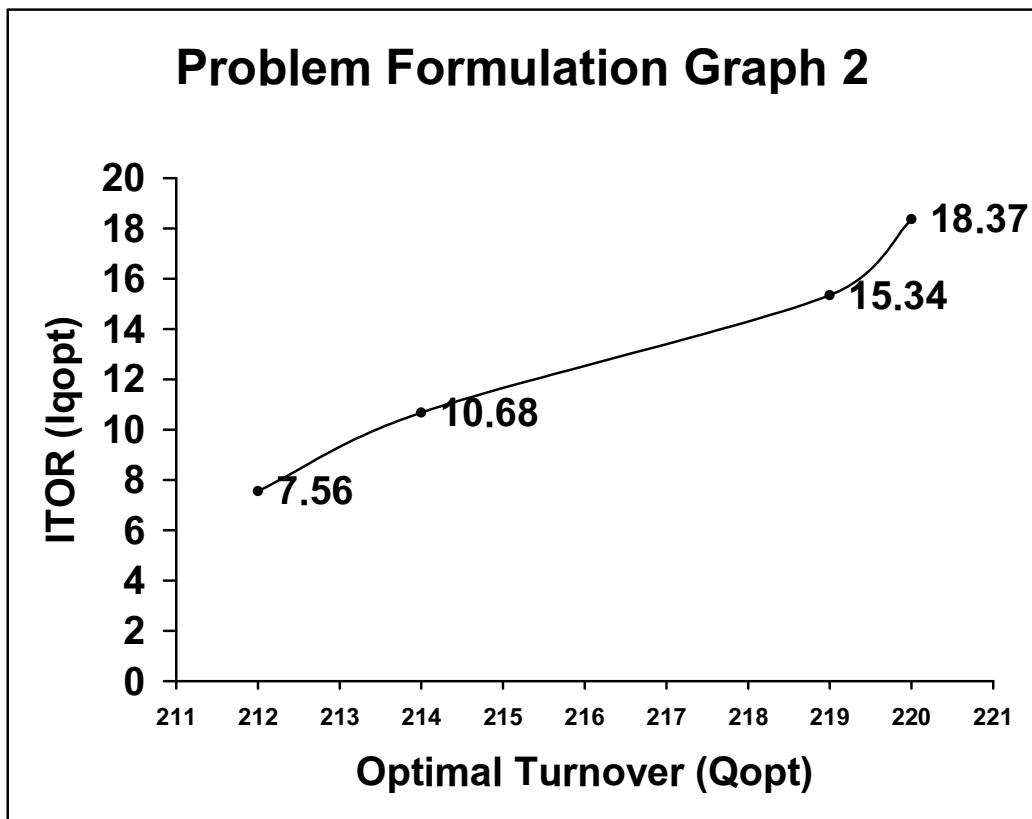
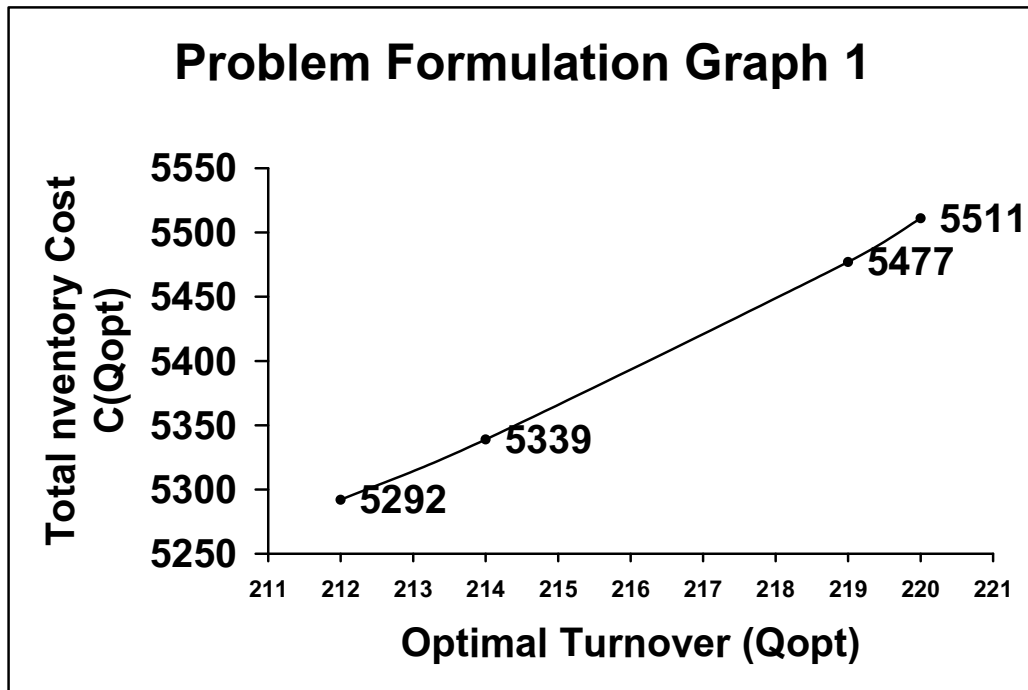
Here, β_1 increase and β_2 decreases with increase in demand. Here the order quantity is maintained with minimum total inventory cost. Also ITOR increases which shows that the goodwill of the customer is taken care off.

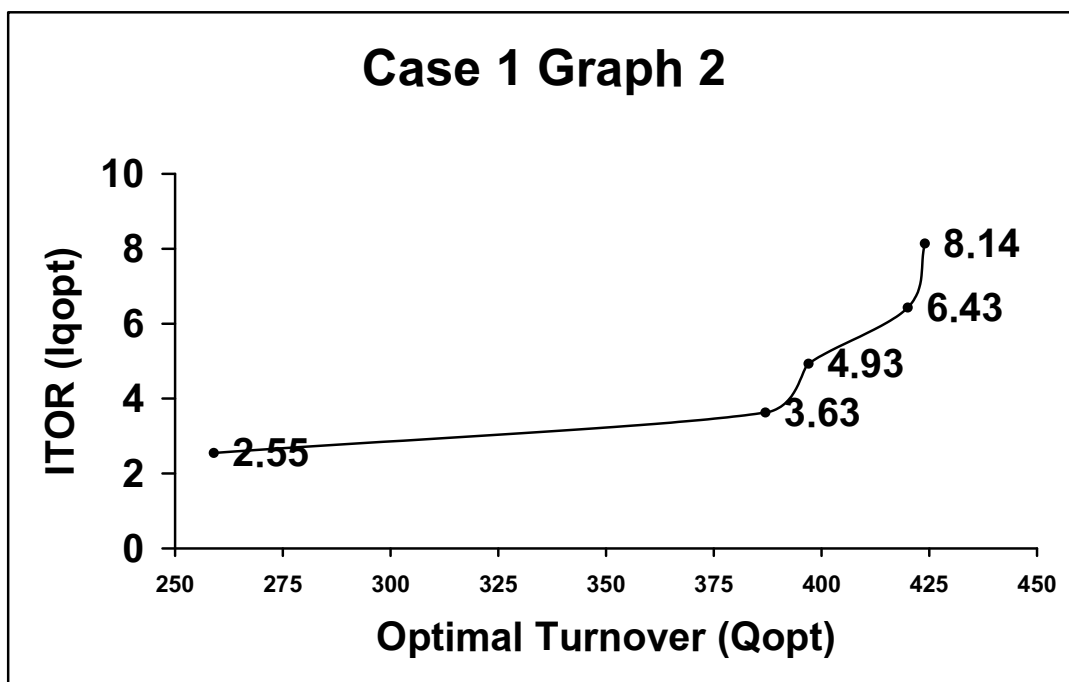
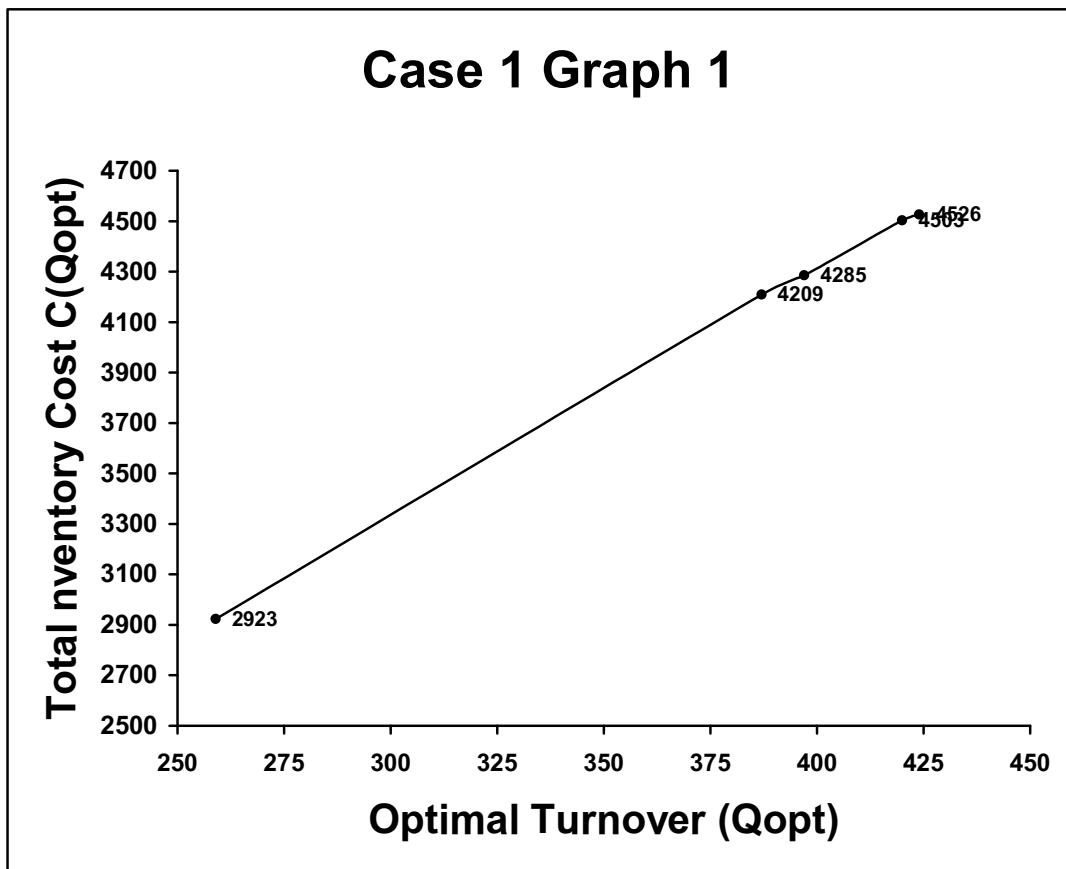
Conclusion of the Paper:

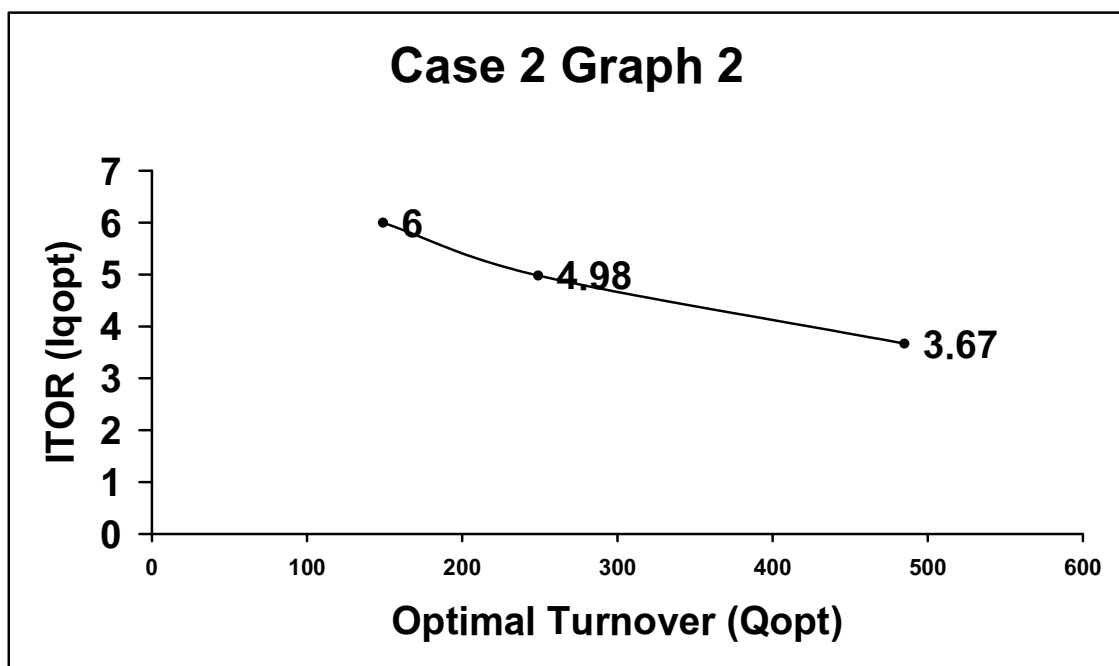
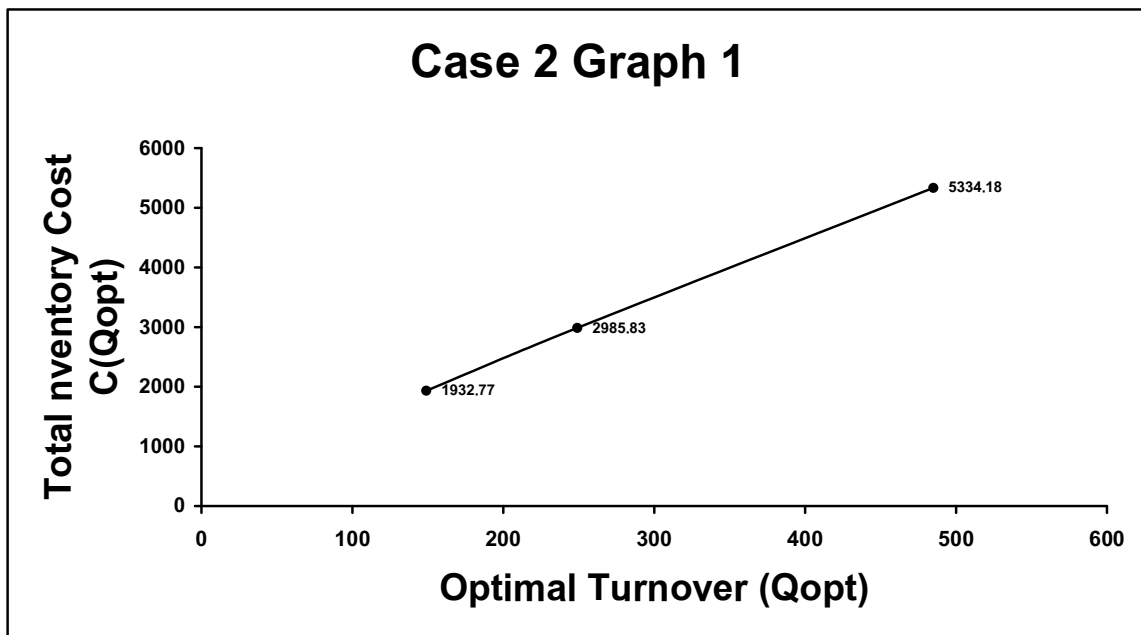
Financial Analysis reveals that the turnover plays a very important role in the overall performance of a firm. A low or high turnover is an indication of poor or good management. A low turnover implies too much of inventory being held as obsolete and a high turnover imply incurring shortages, there by losing the good will of the customers. Thus, usually a balance is struck between these two turnovers.

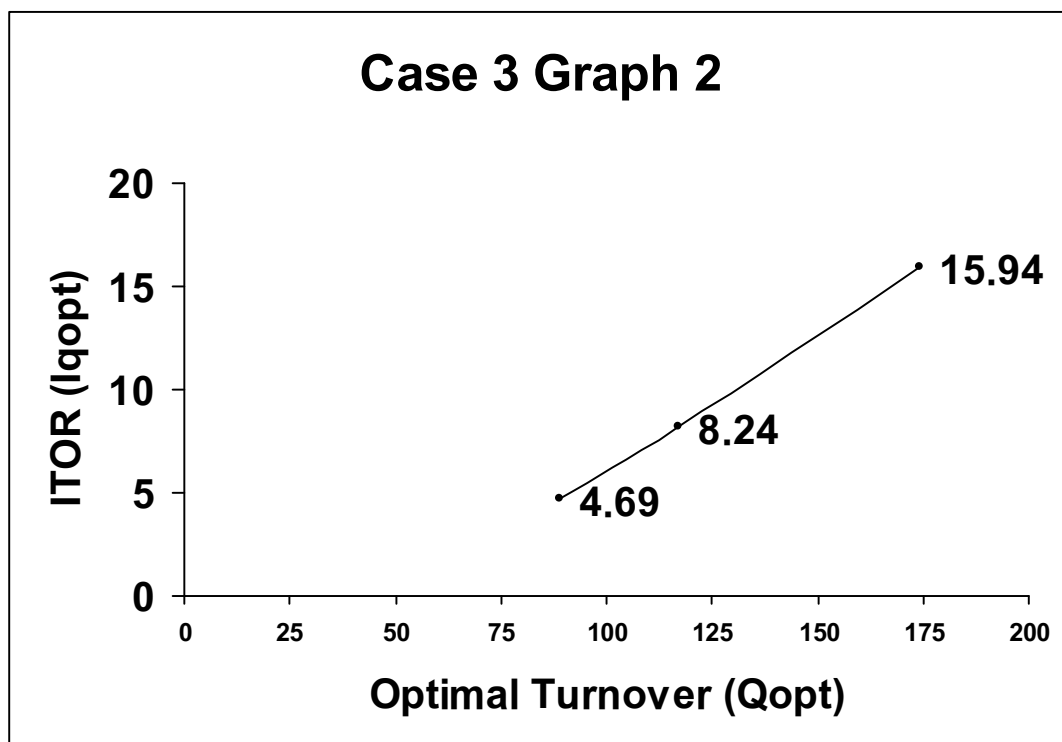
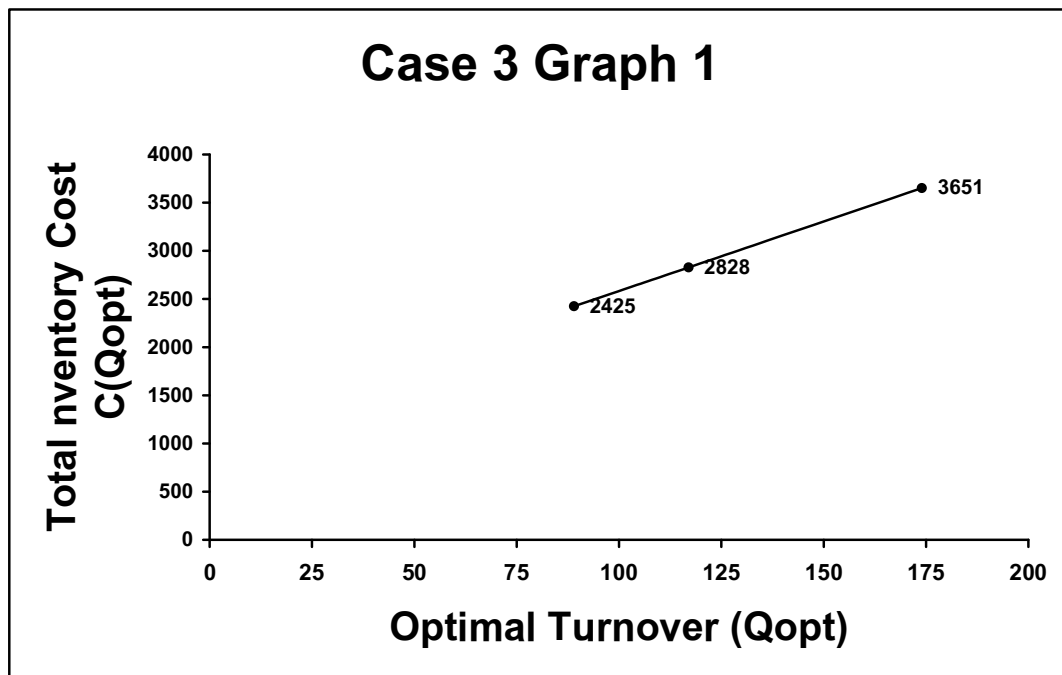
In this paper, the problem of turnovers has been considered under the ITOR Schedule where in the units are supplied according to the demand, there by eliminating the waste. Using this analogy the ideal situation of ITOR production and supply is undertaken. The model studied in this paper, it is unanimously proved that a high turnover is very beneficial to the firm under the ITOR schedule because shortages are not permitted. Thus, an overall review of the model formulated reveal that high turnover is an indication of good management leading to the better performance of the firm. Moreover the risk of large stock outs is minimized.

Appendix –I:









Appendix – II:

Program 1: Problem Formulation

```
/*
Program for
Problem Formulation
*/
#include <stdio.h>
#include <math.h>
#include <conio.h>
FILE *fp;
float d, c1, c3, p, qopt, cqopt, iqopt;
int n, sample;

main()
{
    clrscr();
    if ((fp=fopen("ch2m1pf.dat", "rt"))==NULL)
    {
        printf("Data file not found or cannot be opened");
        getch();
        clrscr();
        return (0);
    }
    fscanf(fp, "%d", &sample);
    printf("\nProblem Formulation\n\n");
    printf("Total Samples %d\n", sample);
    printf("\n*****");
    printf("\nItem    Demand    Holding    Ordering    Purchase    Optimal    Total
Optimal ");
    printf("\n          (Units)    Cost        Cost        Cost        Turnover
Inventory Turnover");
    printf("\n          D          C1          C3          p          (Units)    Cost
Ratio ");
    printf("\n                                   Qopt    C(Qopt)
I(Qopt) ");
    printf("\n*****");
    for (n=1; n<=sample; n++)
    {
        fscanf(fp, "%f %f %f %f", &d, &c1, &c3, &p);
        qopt=pow(2*d*c3/c1, 0.5);
        cqopt=qopt*c1/2+d*c3/qopt;
        iqopt=d*p/pow(2*d*c1*c3, 0.5);
        printf("\n%3d    %5.0f    %5.0f    %5.0f\t %5.2f\t %5.0f
%5.0f    %5.2f", n, d, c1, c3, p, qopt, cqopt, iqopt);
    }
    printf("\n*****");
    fclose(fp);
    return (0);
}
```

Program 2: Case - I

```
/*
Program for
Case 1
*/
#include <stdio.h>
#include <math.h>
#include <conio.h>
FILE *fp;
float d,c1,c3,p,qopt,cqopt,iqopt,alpha,bita,delta;
int n,sample;

main()
{
    clrscr();
    if ((fp=fopen("ch2m1c1.dat","rt"))==NULL)
    {
        printf("Data file not found or cannot be opened");
        getch();
        clrscr();
        return (0);
    }
    fscanf(fp,"%d",&sample);
    printf("\nCase I \n\n");
    printf("Total Items %d\n",sample);
    printf("\n*****");
    printf("\nItem Demand Holding Ordering Purchase Alpha Bit a Optimal
Total Inventory");
    printf("\n (Units) Cost Cost Cost Turnover
Inventory Turnover ");
    printf("\n D C1 C3 p (Units)
Cost Ratio ");
    printf("\n Qopt
C(Qopt) I(Qopt) ");
    printf("\n*****");
    for (n=1; n<=sample; n++)
    {
        fscanf(fp,"%f %f %f %f %f",&c1,&c3,&p,&alpha,&bita);
        delta=pow((8*float(pow(alpha,3))*pow(p,2)*c1*c3),0.5);
        qopt=(-(4*alpha*bita*p*c3)+delta)/(4*pow(bita,2)*p*c3-
2*alpha*p*c1);
        if (qopt<0)
            qopt=((-4*alpha*bita*p*c3)-
delta)/(4*pow(bita,2)*p*c3-2*alpha*p*c1);
        d=alpha+bita*qopt;
        cqopt=qopt*c1/2+d*c3/qopt;
        iqopt=d*p/cqopt;
        printf("\n%3d %5.0f %5.2f %5.2f %5.2f %5.0f %4.0f
%5.0f %5.0f %5.2f",n,d,c1,c3,p,alpha,bita,qopt,cqopt,iqopt);
    }
    printf("\n*****");
}
```

```
        fclose(fp);
        return (0);
    }
```

Program 3: Case - II

```
/*
Program for
Case - 2
*/
#include <stdio.h>
#include <math.h>
#include <conio.h>
#include <stdlib.h>
FILE *fp;
float d,c1,c3,p,cqopt,iqopt,bita,x;
int n,sample,itrs;
double a1,a2,a3,q1,q2,q3,qopt;
inline float f(float q)
{
    return (2*c3*bita*pow(q,x)-c1*(1-x)*q*q);
}
main()
{
    clrscr();
    if ((fp=fopen("ch2mlc2.dat","rt"))==NULL)
    {
        printf("Data file not found or cannot be opened");
        getch();
        clrscr();
        return (0);
    }
    fscanf(fp,"%d",&sample);
    printf("\nCase II \n\n");
    printf("Total Items %d\n",sample);
    printf("\n*****");
    printf("\nItem Demand Holding Ordering Purchase x Bita Optimal
Total Inventory");
    printf("\n (Units) Cost Cost Cost Turnover
Inventory Turnover ");
    printf("\n D C1 C3 p (Units)
Cost Ratio ");
    printf("\n Qopt
C(Qopt) I(Qopt) ");
    printf("\n*****");
    for (n=1; n<=sample; n++)
    {
        fscanf(fp,"%f %f %f %f %f",&c1,&c3,&p,&x,&bita);
        q1=1;
        q2=500;
        itrs=1;
        a1=f(q1);
        a2=f(q2);
```

```
        if (a1*a2>0)
        {
            printf("\nInitial approximations are
unsuitable\n");
            exit(1);
        }
        do
        {
            itrs=fabs(a2-a1);
            q3=(q1*a2-q2*a1)/(a2-a1);
            a3=f(q3);
            if (a1*a3<0)
            {
                q2=q3;
                a2=a3;
            }
            else
            {
                q1=q3;
                a1=a3;
            }
        }
        while (fabs((double)q1-q2)>0.0001 && (f(q3) != 0));
        qopt=q3;
        d=bita*pow(qopt,x);
        cqopt=qopt*c1/2+d*c3/qopt;
        iqopt=d*p/cqopt;
        printf("\n%3d %5.0f %5.0f %5.0f %5.2f %5.2f %4.0f
%5.0f %7.2f %5.2f",n,d,c1,c3,p,x,bita,qopt,cqopt,iqopt);
    }
    printf("\n*****");
    fclose(fp);
    return (0);
}
```

Program 4: Case - III

```
/*
Program for
Case 3
*/
#include <stdio.h>
#include <math.h>
#include <conio.h>
FILE *fp;
float d,c1,c3,p,qopt,cqopt,iqopt,bta1,bta2,delta;
float a,b;
int n,sample;

main()
{
    clrscr();
    if ((fp=fopen("ch2m1c3.dat","rt"))==NULL)
    {
```

```
printf("Data file not found or cannot be opened");
getch();
clrscr();
return (0);
}
fscanf(fp, "%d", &sample);
printf("\nCase III \n\n");
printf("Total Items %d\n", sample);
printf("\n*****");
*****
printf("\nItem Demand Holding Ordering Purchase Bit1 Bit2 Optimal
Total Inventory");
printf("\n (Units) Cost Cost Cost Turnover
Inventory Turnover ");
printf("\n D C1 C3 p (Units)
Cost Ratio ");
printf("\n Qopt
C(Qopt) I(Qopt) ");
printf("\n*****");
*****
for (n=1; n<=sample; n++)
{
    fscanf(fp, "%f %f %f %f %f", &c1, &c3, &p, &bta1, &bta2);
    delta=8*pow(p, 2)*pow(bta1, 2)*bta2*c1*c3;
    a=2*p*pow(bta2, 2)*c3 - p*bta2*c1;
    b=-4*p*bta1*bta2*c3;
    // qopt= (-b+pow(delta, 0.5))/(2*a);
    qopt=(-b-pow(delta, 0.5))/(2*a);
    d=bta1*qopt - bta2*pow(qopt, 2);
    cqopt=qopt*c1/2+d*c3/qopt;
    iqopt=d*p/cqopt;
    printf("\n%3d %5.0f %5.0f %5.0f %5.2f %5.0f %4.1f
%5.0f %5.0f %5.2f", n, d, c1, c3, p, bta1, bta2, qopt, cqopt, iqopt);

}
printf("\n*****");
*****
fclose(fp);

return (0);

}
```

REFERENCES:

1. Ackoff R.L. & Sasieni M.W. (1984), Fundamentals of Operations Research, John Wiley and Sons Inc., New York
2. Aggrawal S.P. and Ayalwadi D.R. (1988), Optimum Ordering Policy Under Varying Price and Promotional Effort, International Journal of Management and System, Vol-4, No-3, Sept-Dec, 1988, ISSN 0970-0439
3. Bass F.M. and Buzull D.R. (1961), Mathematical Models and Methods in marketing, Artical by H.D.Miles, A study of promotional Competition, Homewood Richard O Orwin Inc., New York

4. Charles T.H. (1981), Introduction to Financial Accounting, Prentice Hall, New York
5. Cutris, Edward T. (1962), Company Organization of the Financial Function, Research Study, American Management Association, Inc. New York, No. 55
6. Foulke, Roy A. (1960), Inventories and Business Health, Dun & Bradstreet Publications, Inc., New York
7. Gupta S.K and Krishnan K.S. (1967), Mathematical Models in Marketing, Operations Research, vol-15 no.-6 page 1040-1050
8. Hall R.W. & Dow Jones (1983), Zero Inventories, Irwin, New York
9. Jeya Chandra M. and M.L. Bahner (1985), The effects of inflation and the time value of money on some inventory systems, International Journal of production research, Vol. 23 No. 4 Page 723-730
10. Kanti swarup, P.K.Gupta, Man Mohan (1978), Operation Research, Sultan Chand and Sons, New Delhi
11. Kishore R.M. (2002), Financial Management, Taxman
12. Kun-Sen Chang, Yung-Fa Huang (2004), Optimal replenishment policies for EOQ Inventory Model with Limited Storage Capacity under permissible delay in payments, OPSEARCH, Vol. 41 No. 1 Page 16-34
13. Li Zhuang (1994), Towards a more economic production and Just-In-Time delivery System, International Journal of Production Research, Vol-36 page 307-313
14. Pathak B.V. (2003), Indian Financial System, Pearson Education
15. Philip Kotler (1971), Marketing Decision Making: A Model Building Approach, Holt Rinehart and Winston Inc.
16. Prakash L.Abad (1988), Determining optimum Selling Price and Lot Size when the Supplier Offers all-unit Quantity discount, Decision Sciences, Vol-19, No. 3
17. Prasanna Chandra (1997), Financial Management: Theory and Practice, Tata McGraw-Hill
18. Sarker B.R., Jamal A.M.M., Wang S. (2001), Optimal payment time under permissible delay in payment for products with deterioration, Production planning and control, Vol. 11 Page 380-390
19. Subramanyam E and Kumarswamy S (1981), EOQ formula under varying marketing policies and conditions, AIIE Transactions, vol-13, page 312-324
20. Tang J.T. (2002), On the economic order quantity under condition of permissible delay in payments, Journal of the operational research society, Vol. 53 Page 915-918